Engineering Casimir interactions with metamaterials

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Outline of this Talk

- Casimir forces and metamaterials
  - What is a metamaterial?
  - Proposals for Casimir force manipulation with metamaterials
  - Is Casimir repulsion possible?
- Some very new results (paper under review)
  - Stay till the end....

Metamaterials and Casimir

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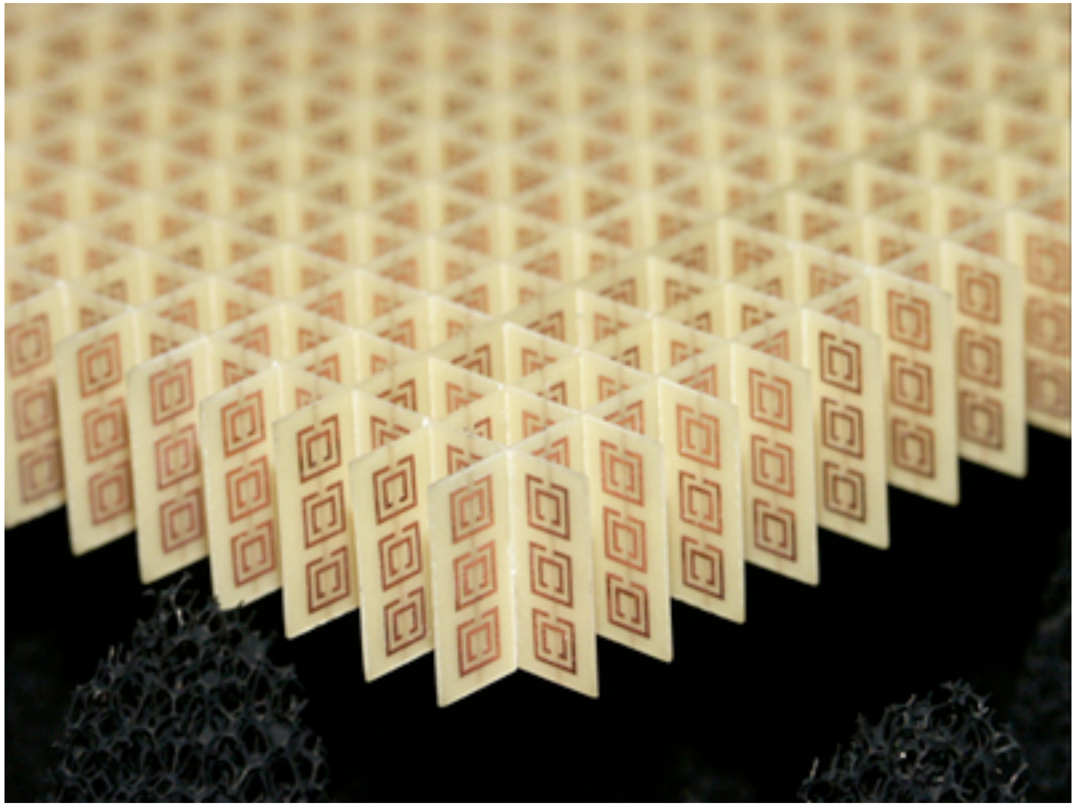
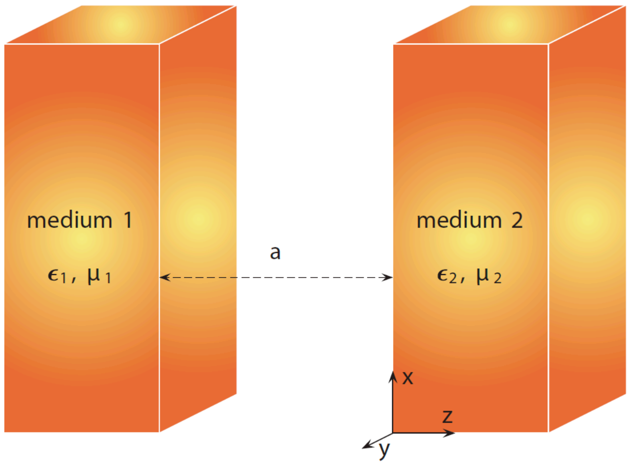


Effects of materials

The Lifshitz formula: Lifshitz (1956)

$$\boxed{\frac{F}{A} = 2\hbar\,\mathrm{Im}\int_0^\infty \frac{d\omega}{2\pi}\int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr}\frac{\mathbf{R}_1\cdot\mathbf{R}_2 e^{2iK_3 d}}{1-\mathbf{R}_1\cdot\mathbf{R}_2 e^{2iK_3 d}}}$$



$$K_3 = \sqrt{\omega^2/c^2 - k_\parallel^2}$$

Reflection matrices (Fresnel formulas for isotropic media):

$$r^{\mathrm{TM,TM}}(\omega,\mathbf{k}_\parallel) = \frac{\epsilon(\omega)K_3 - \sqrt{\epsilon(\omega)\mu(\omega)\omega^2/c^2 - k_\parallel^2}}{\epsilon(\omega)K_3 + \sqrt{\epsilon(\omega)\mu(\omega)\omega^2/c^2 - k_\parallel^2}}$$

$r^{\mathrm{TE,TE}} = r^{\mathrm{TM,TM}}$ **with** $\epsilon \leftrightarrow \mu$

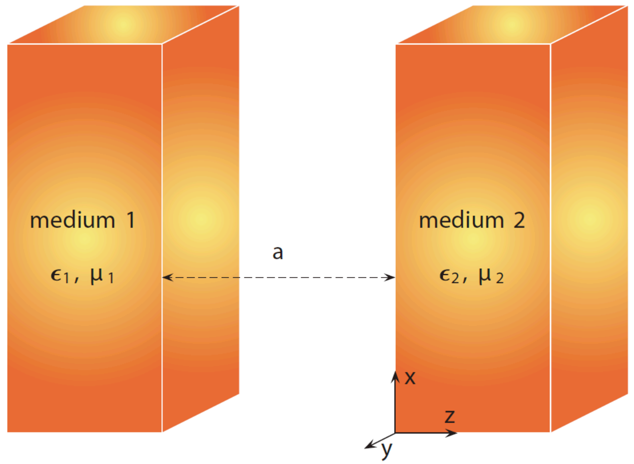
Relevant frequencies:

$$\left.\begin{array}{lll} \epsilon(\omega) \xrightarrow{\omega\gg\Omega_p} 1 & \Rightarrow & r^{p,p}\approx 0 \text{ (Transparent plates)} \\ \omega \gg c/d & \Rightarrow & e^{2iK_3 d}\approx 0 \text{ (Fast oscillations)} \end{array}\right\} \Rightarrow F\approx 0$$

$$\boxed{0 \le \omega \le \min\{\Omega_p, c/d\}}$$

Going to imaginary frequencies

$$\frac{F}{A} = 2\hbar \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}$$



Kramers-Kronig (causality) relations:

$$\epsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \epsilon''(\omega)}{\omega^2 + \xi^2} d\omega \qquad \mu(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \mu''(\omega)}{\omega^2 + \xi^2} d\omega$$

Dominant frequencies below the near-infrared/optical region of the EM spectrum (gaps d= 200-1000 nm)

The important message is that Casimir is a broad-band frequency phenomenon

The sign of the Casimir force

$$\frac{F}{A} = 2\hbar \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} K_3 \mathrm{Tr} \frac{\mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}{1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2K_3 d}}$$

The sign of the force is directly connected to the sign of the product of the reflection coefficients on the two plates, evaluated at imaginary frequencies. As a rule of thumb, we have (p=TE, TM)

$$R_1^p(i\xi) \cdot R_2^p(i\xi) > 0 \ (\forall\ \xi \leq c/d) \Rightarrow \text{Attraction}$$

$$R_1^p(i\xi) \cdot R_2^p(i\xi) < 0 \ (\forall\ \xi \leq c/d) \Rightarrow \text{Repulsion}$$

In terms of permittivities and permeabilities:

$$\epsilon_a(i\xi) \gg \epsilon_b(i\xi)$$

$$\mu_b(i\xi) \gg \mu_a(i\xi)$$

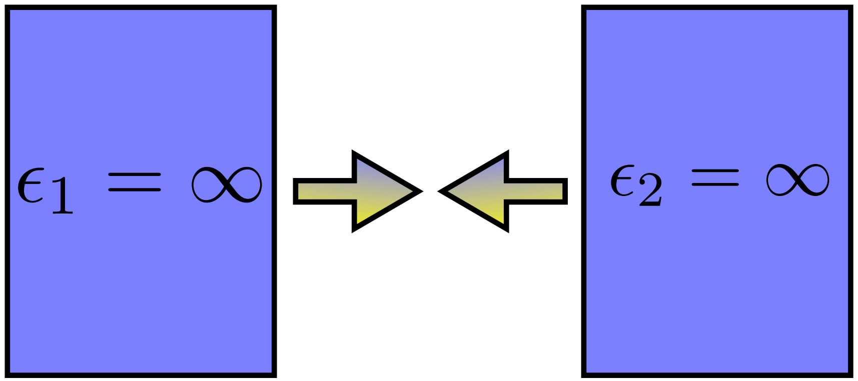
$\longrightarrow$ Repulsion

Ideal attraction-repulsion

- **Ideal attractive limit**

Casimir (1948)

$$\boxed{\frac{F}{A} = +\frac{\pi^2}{240}\,\frac{\hbar c}{d^4}}$$



Ideal attraction-repulsion

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- **Ideal attractive limit**

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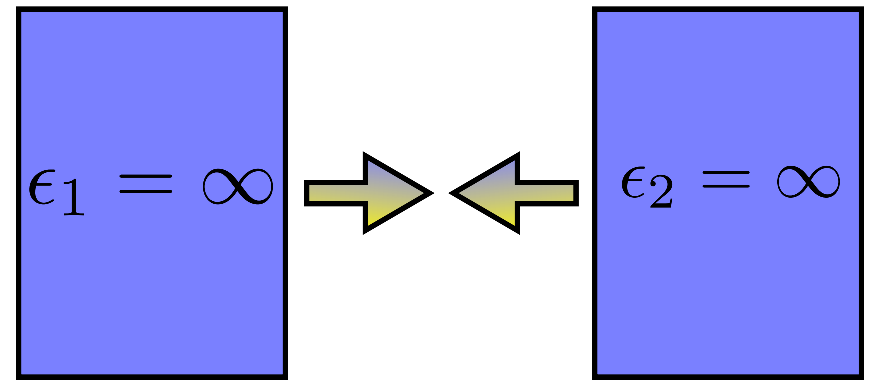
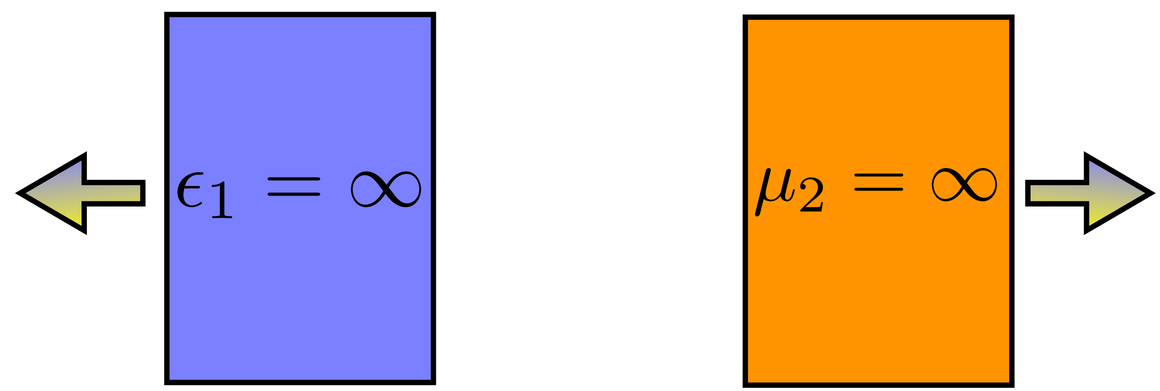


- **Ideal repulsive limit**

  Boyer (1974)

$$\boxed{\frac{F}{A} = -\frac{7}{8}\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



Ideal attraction-repulsion

- ## Ideal attractive limit

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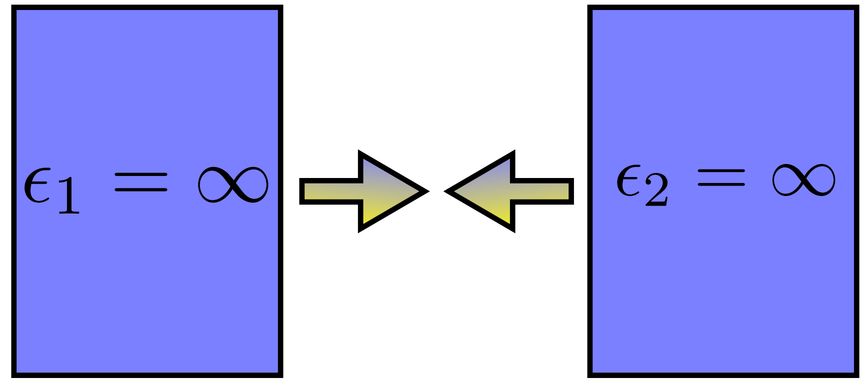

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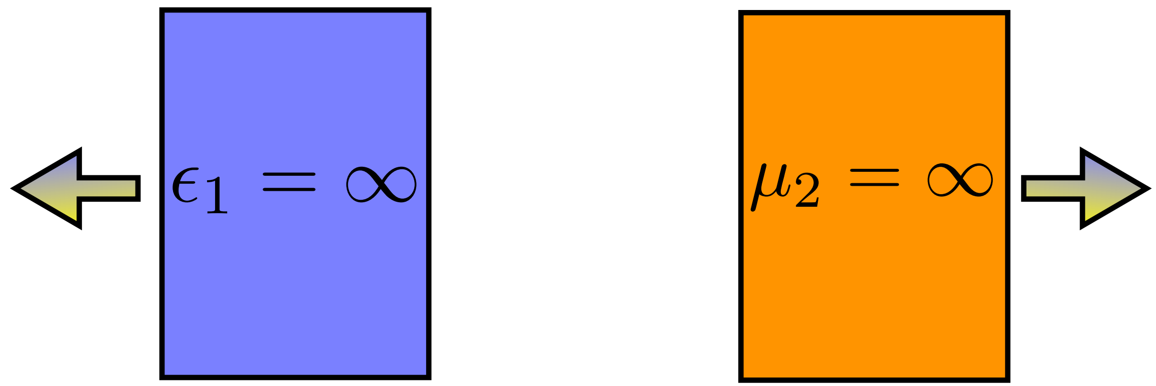

- ## Real repulsive limit

Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$

Ideal attraction-repulsion

- ## Ideal attractive limit

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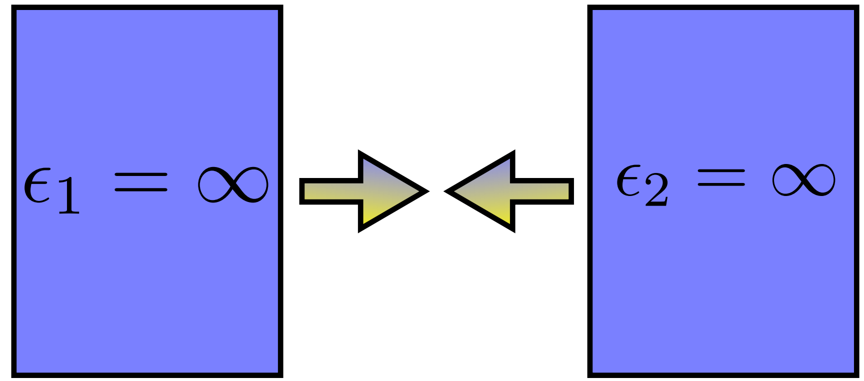

- ## Ideal repulsive limit

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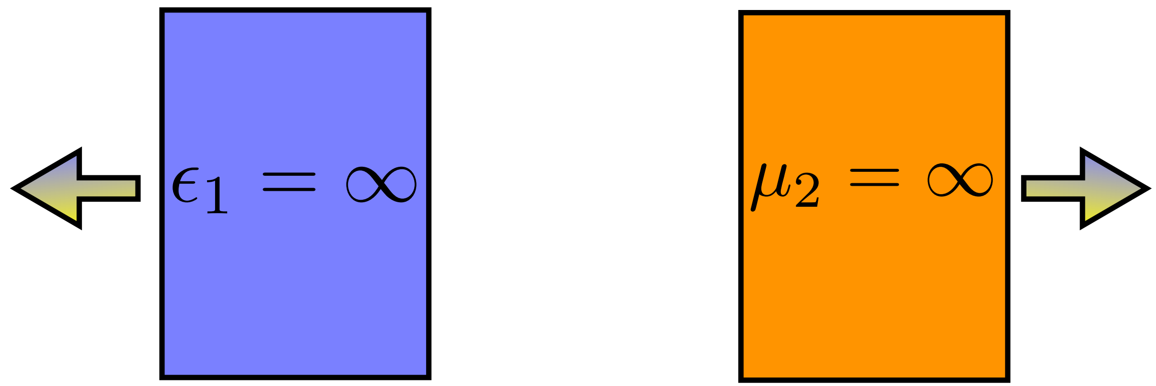

- ## Real repulsive limit

Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$ $\longrightarrow$ Metamaterials

Quantum levitation with MMs?

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Physicists have 'solved' mystery of levitation

By Roger Highfield, Science Editor
Last Updated: 1:41am BST 06/08/2007

Levitation has been elevated from being pure science fiction to science fact, according to a study reported today by physicists.

In earlier work the same team of theoretical physicists showed that invisibility cloaks are feasible.

Now, in another report that sounds like it comes out of the pages of a Harry Potter book, the University of St Andrews team has created an 'incredible levitation effects' by engineering the force of nature which normally causes objects to stick together.

Professor Ulf Leonhardt and Dr Thomas Philbin, from the University of St Andrews in Scotland, have worked out a way of reversing this phenomenon, known as the Casimir force, so that it repels instead of attracts.

Their discovery could ultimately lead to frictionless micro-machines with moving parts that levitate But they say that, in principle at least, the same effect could be used to levitate bigger objects too, even a person.

In theory the discovery could be used to levitate a person

The Casimir force is a consequence of quantum mechanics, the theory that describes the world of atoms and subatomic particles that is not only the most successful theory of physics but also the most baffling.

The force is due to neither electrical charge or gravity, for example, but the fluctuations in all-pervasive energy fields in the intervening empty space between the objects and is one reason atoms stick together, also explaining a "dry glue" effect that enables a gecko to walk across a ceiling.

Now, using a special lens of a kind that has already been built, Prof Ulf Leonhardt and Dr Thomas Philbin report in the New Journal of Physics they can engineer the Casimir force to repel, rather than attract.

Because the Casimir force causes problems for nanotechnologists, who are trying to build electrical circuits and tiny mechanical devices on silicon chips, among other things, the team believes the feat could initially be used to stop tiny objects from sticking to each other.

Prof Leonhardt explained, "The Casimir force is the ultimate cause of friction in the nano-world, in particular in some microelectromechanical systems.

Such systems already play an important role - for example tiny mechanical devices which triggers a car airbag to inflate or those which power tiny 'lab on chip' devices used for drugs testing or chemical analysis.

Micro or nano machines could run smoother and with less or no friction at all if one can manipulate the force." Though it is possible to levitate objects as big as humans, scientists are a long way off developing the technology for such feats, said Dr Philbin.

The practicalities of designing the lens to do this are daunting but not impossible and levitation "could happen over quite a distance".

Prof Leonhardt leads one of four teams - three of them in Britain - to have put forward a theory in a peer-reviewed journal to achieve invisibility by making light waves flow around an object - just as a river flows undisturbed around a smooth rock.

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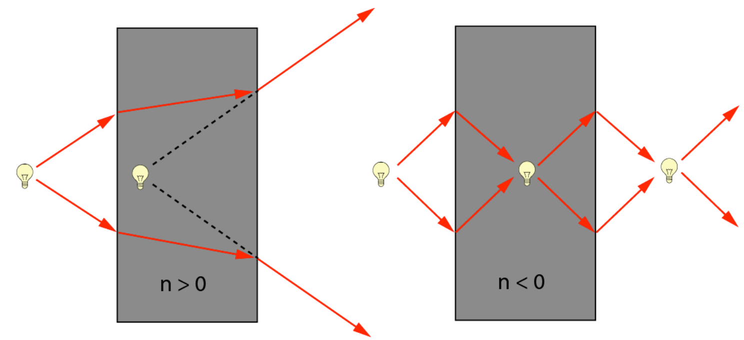
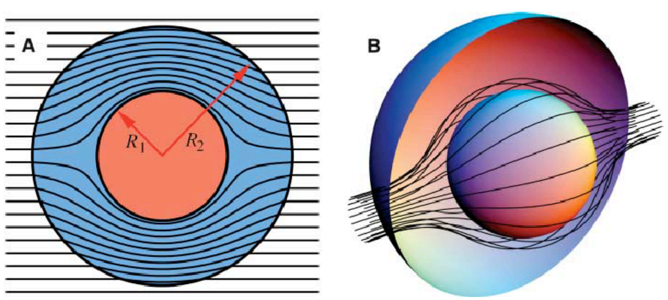
"In theory the discovery could be used to levitate a person"

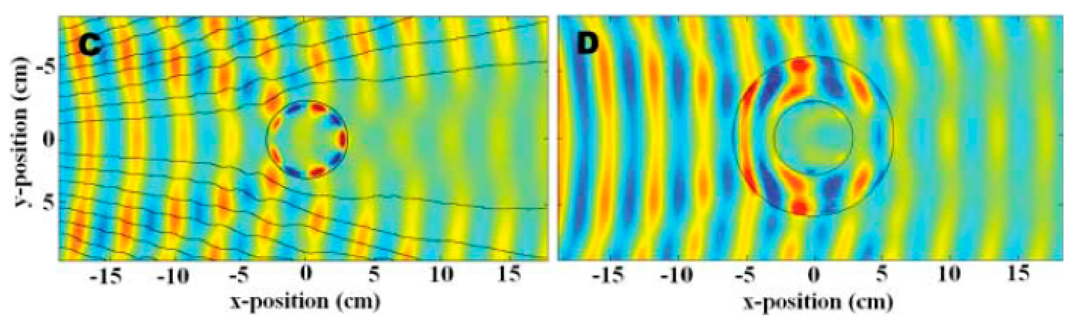
Metamaterials

- Artificial structured composites with designer electromagnetic properties
- MMs are strongly anisotropic, dispersive, magneto-dielectric media.

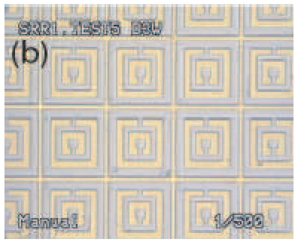
- Negative refraction — Veselago (1968), Smith et al (2000)
- Perfect lens — Pendry (2000)
- Cloaking — Smith et al (2007)

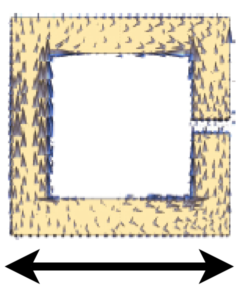




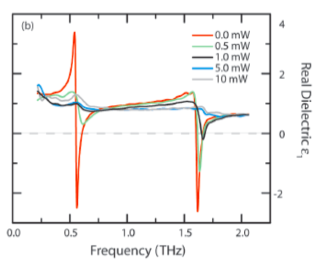


THz MMs: eg split ring resonators



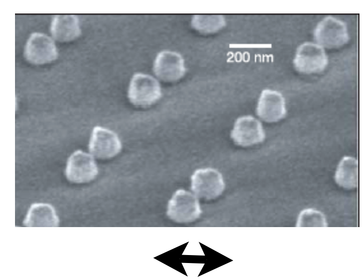


36μm

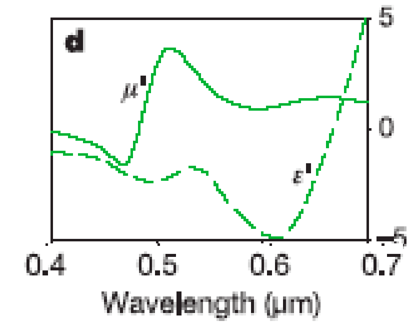


$\epsilon, \mu < 0$

Optical MMs: eg nano-pillars



200nm



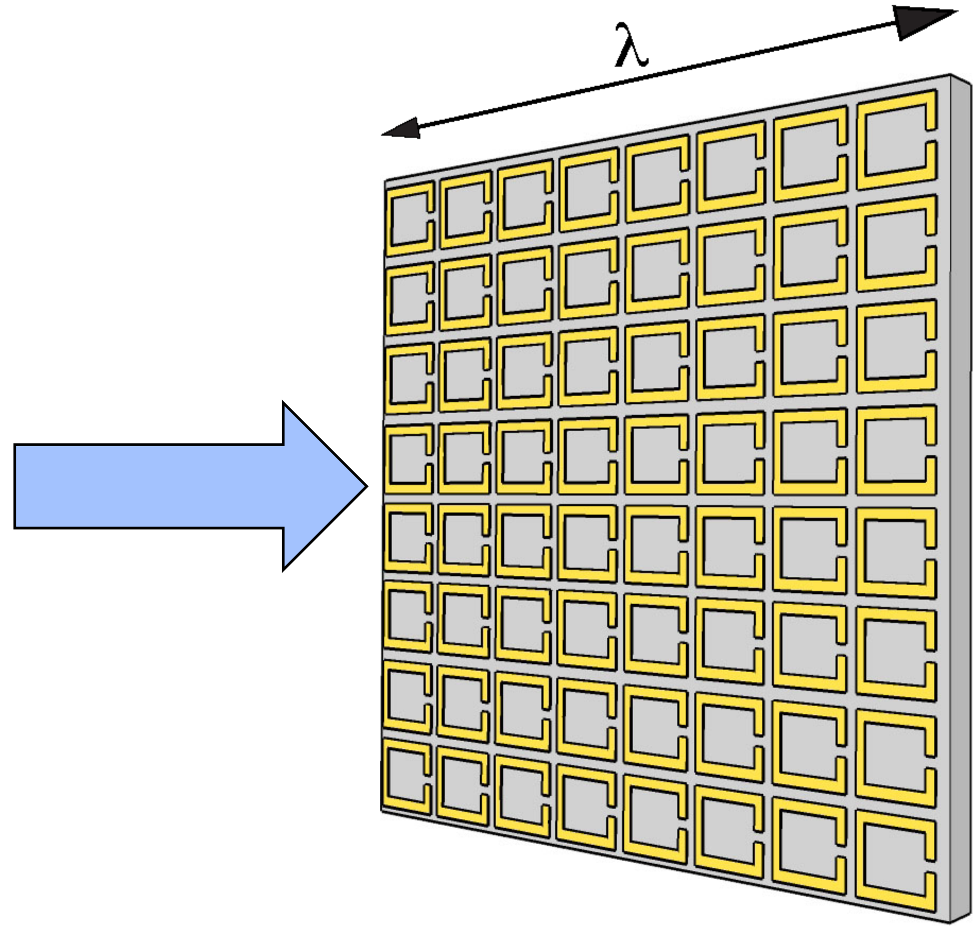
Effective medium approximation

We want to compute the Casimir force between a metallic plate and a MM. Let us assume a metallic plate in is reasonably well described by a Drude response

$$\epsilon_1(\omega) = 1 - \frac{\Omega_e^2}{\omega^2 + i\gamma_e\omega} \qquad \mu_1 = 1$$

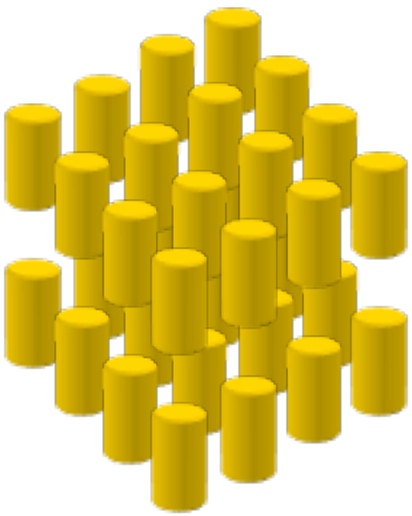
For the MM the optical response is not so simple.....

In the effective medium approximation (EMA) one describes the MM with an effective electric permittivity and an effective magnetic permeability. This is an approximation valid when the MM is probed at wavelengths much larger that the average distance between the constituent "particles" of the MM.

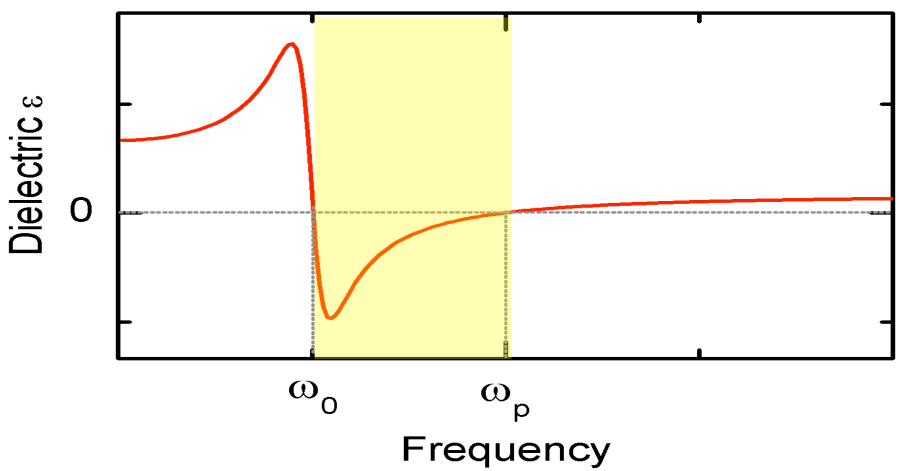


EMA: Electric response

Close to resonance, the optical response can be modeled by a Drude-Lorentz permittivity



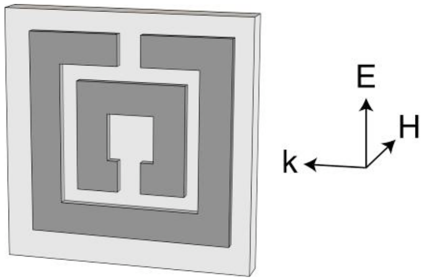
$$\varepsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

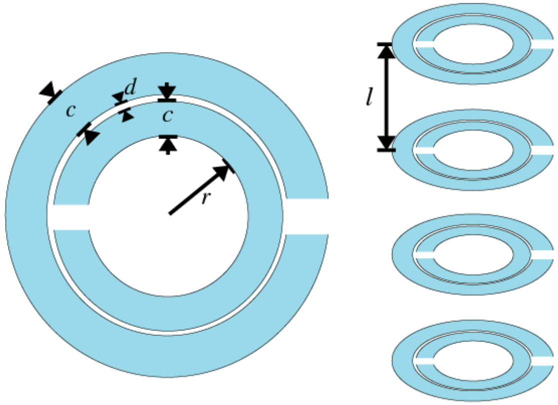


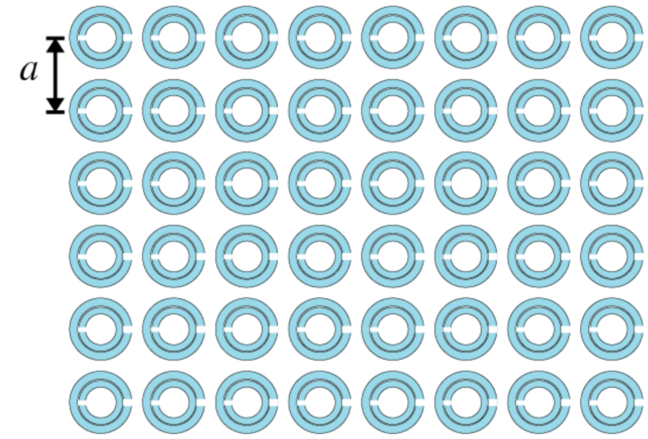
J.B. Pendry ***et al.***, ***Phys. Rev. Lett.*** **76**, 4773 (1996).

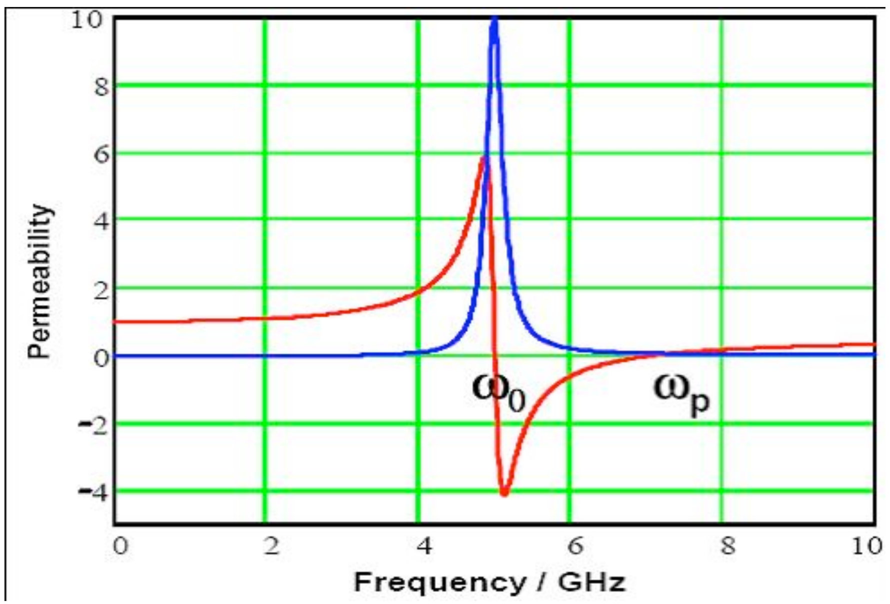
EMA: Magnetic response

$$\mu_{eff} = 1 - \frac{\frac{\pi r^2}{a^2}}{1 + \frac{2\sigma i}{\omega r \mu_0} - \frac{3}{\pi^2 \mu_0 \omega^2 C r^3}}$$









J.B. Pendry *et al.*, *IEEE Trans. Microwave Tech.* **47,** 2075 (1999).

EMA: Drude-Lorentz responses

Close to the resonance, both $\epsilon(\omega)$ and $\mu(\omega)$ can be modeled by Drude-Lorentz formulas

$$\boxed{\begin{aligned}\epsilon_\alpha(\omega) &= 1 - \frac{\Omega_{E,\alpha}^2}{\omega^2 - \omega_{E,\alpha}^2 + i\Gamma_{E,\alpha}\omega} \\ \mu_\alpha(\omega) &= 1 - \frac{\Omega_{M,\alpha}^2}{\omega^2 - \omega_{M,\alpha}^2 + i\Gamma_{M,\alpha}\omega}\end{aligned}}$$

Typical separations

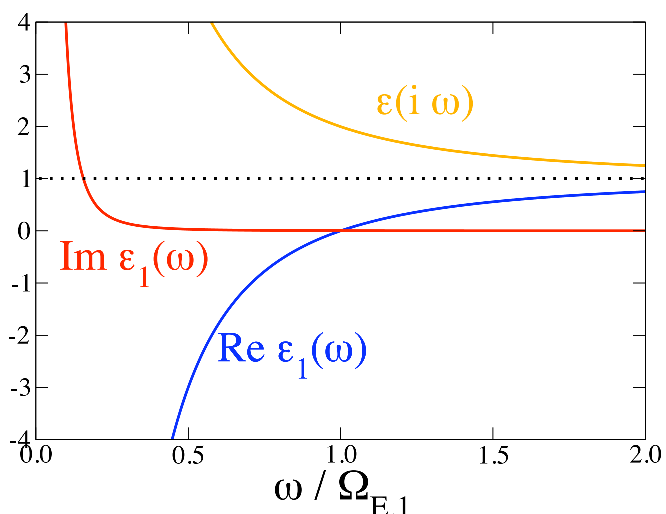
$$d = 200 - 1000 \text{ nm}$$

Infrared-optical frequencies

$$\Omega/2\pi = 5 \times 10^{14}\text{Hz}$$

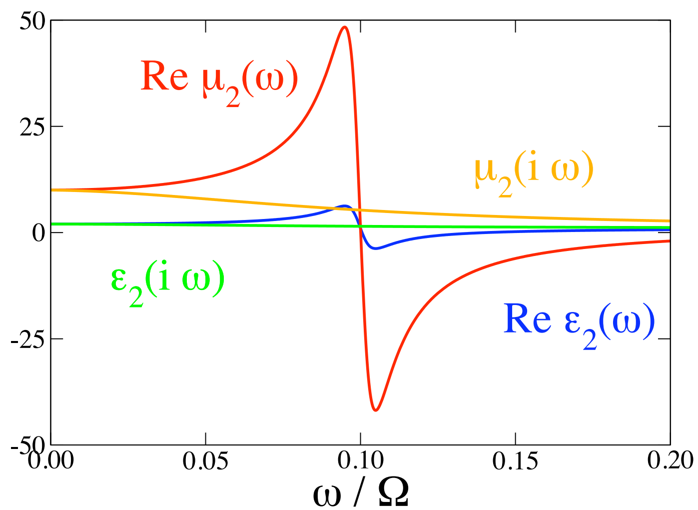
Drude metal (Au)

$\Omega_E = 9.0 \text{ eV} \quad \Gamma_E = 35 \text{ meV}$



Metamaterial

$\text{Re } \epsilon_2(\omega) < 0 \quad \text{Re } \mu_2(\omega) < 0$

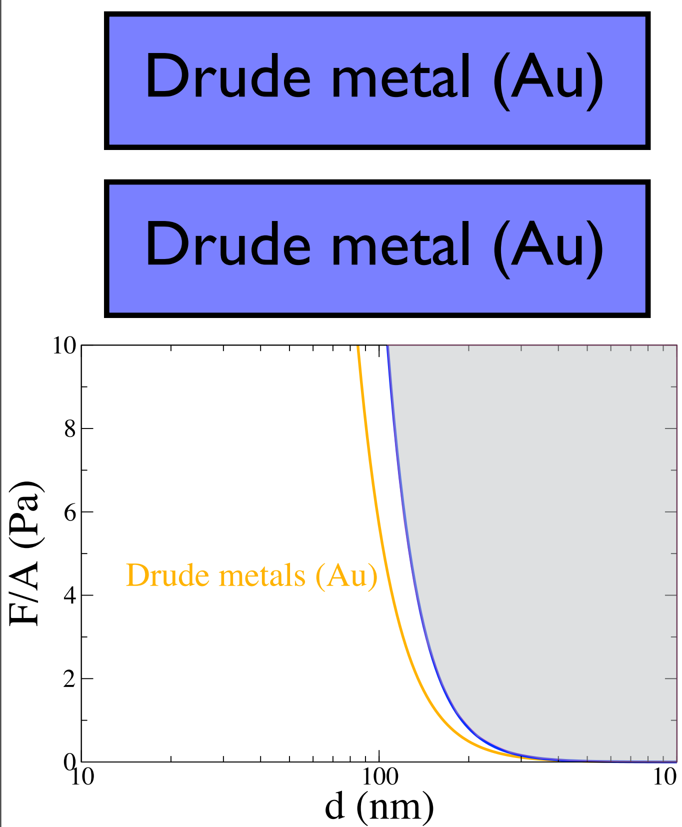


$\Omega_{E,2}/\Omega = 0.1 \quad \Omega_{M,2}/\Omega = 0.3$

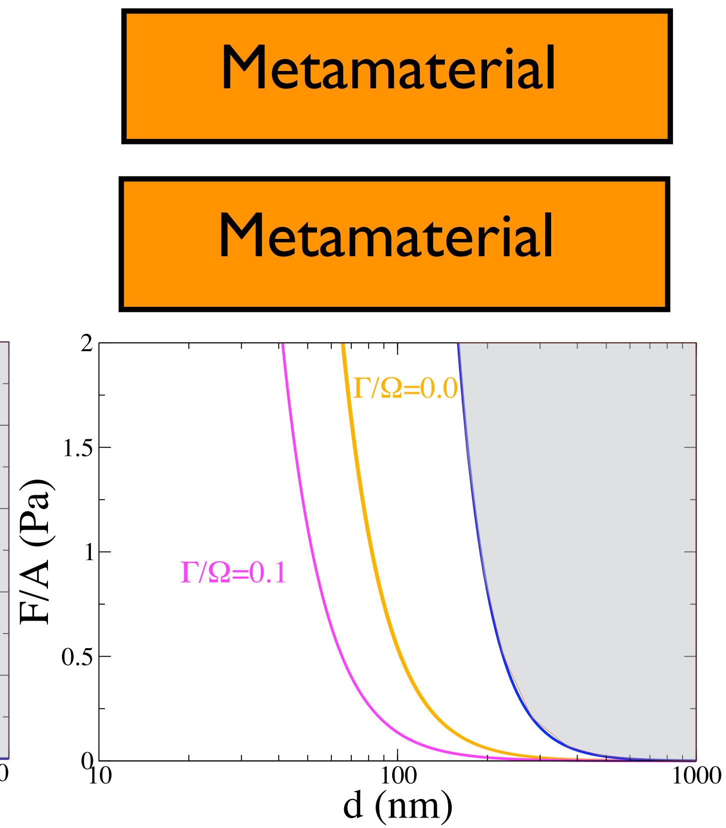
$\omega_{E,2}/\Omega = \omega_{M,2}/\Omega = 0.1$

$\Gamma_{E,2}/\Omega = \Gamma_{M,2}/\Omega = 0.01$

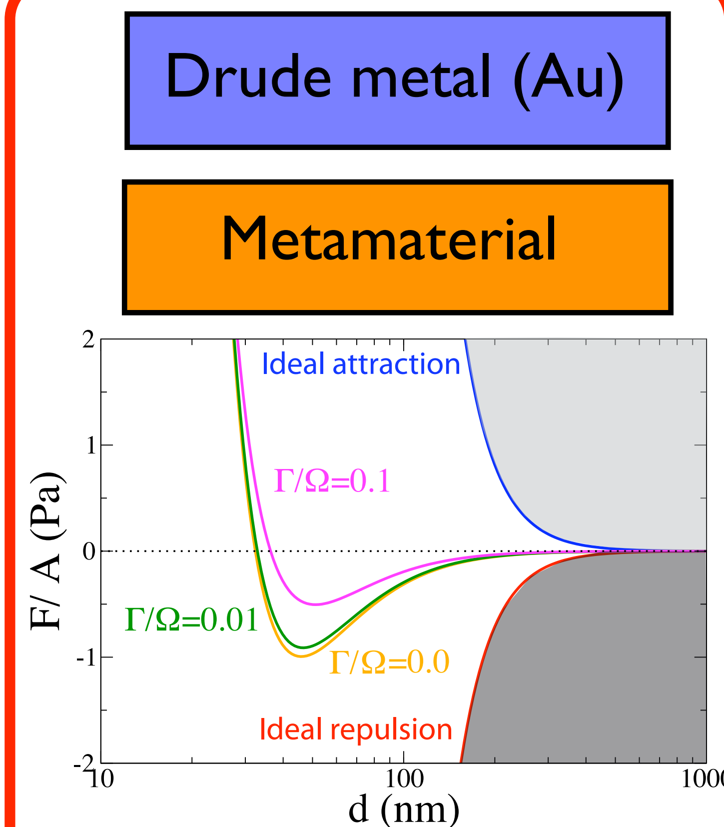
Attraction-repulsion crossover



Only attraction



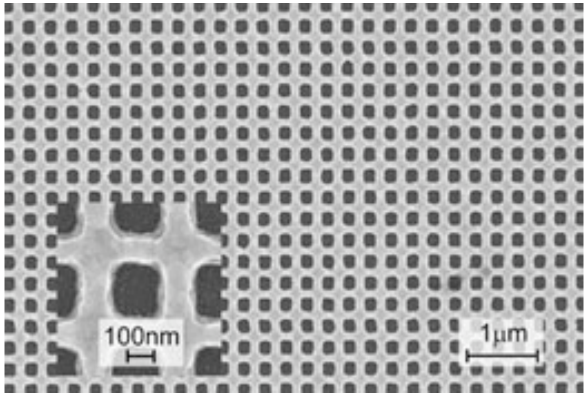
Only attraction



Repulsion-attraction

Drude background

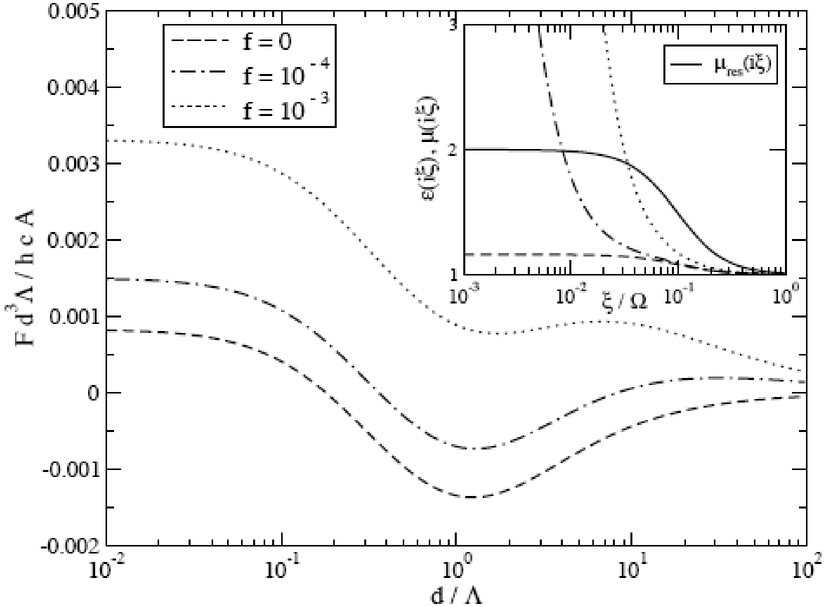
- In some metallic-based MMs, there is a net conductivity due to the metallic structure, like the fishnet design on the right.



$$\epsilon(\omega) = 1 - f\frac{\Omega_D^2}{\omega^2 - i\omega\gamma_D} - (1-f)\frac{\Omega_e^2}{\omega^2 - \omega_e^2 + i\gamma_e\omega}$$

f : filling factor

$$\mu(\omega) = 1 - \frac{\Omega_m^2}{\omega^2 - \omega_m^2 + i\gamma_m\omega}$$



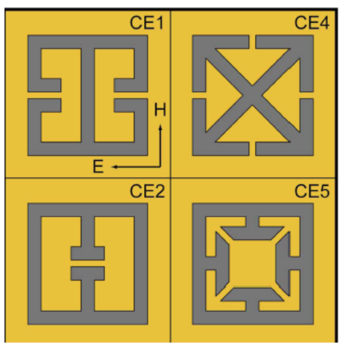
Rosa, DD, Milonni, PRL 2008

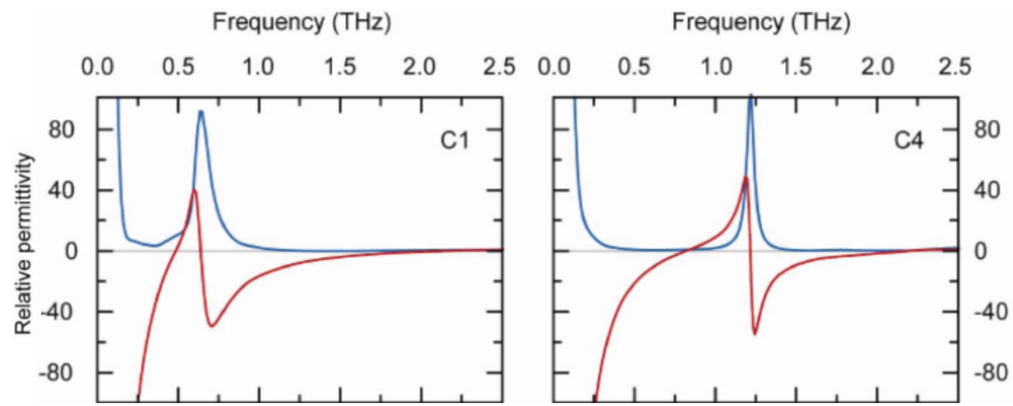
A Drude background is detrimental for Casimir force reduction or repulsion, since it results in an electric response much stronger than the magnetic one

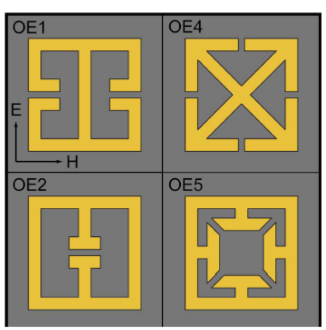
$$\epsilon_2(i\xi) \gg \mu_2(i\xi)$$

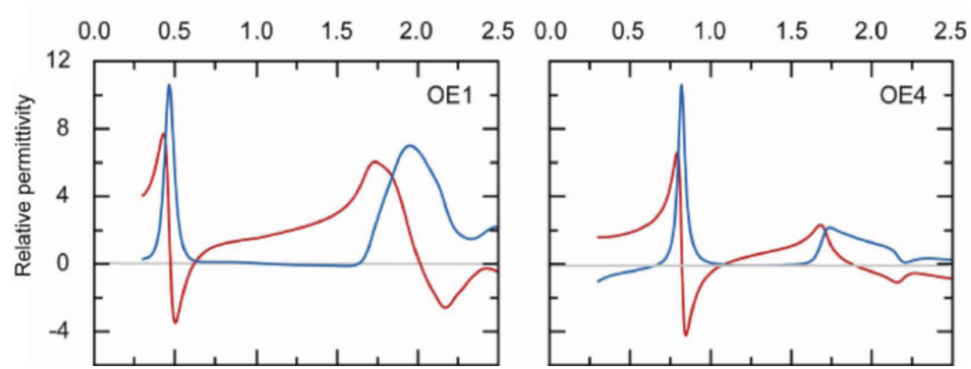
Complementary SRRs + Drude

SRRs structures provide an opportunity to avoid the large Drude background, since they can be built in two natural complementary ways









Optical anisotropy

In an anisotropic medium, the constitutive relations between E, D, B, and H are more involved:

$$\mathbf{D} = \epsilon \cdot \mathbf{E} \qquad \mathbf{H} = \mu^{-1} \cdot \mathbf{B}$$

due to the tensorial nature of the permittivity and permeability

$$\epsilon = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{bmatrix}$$

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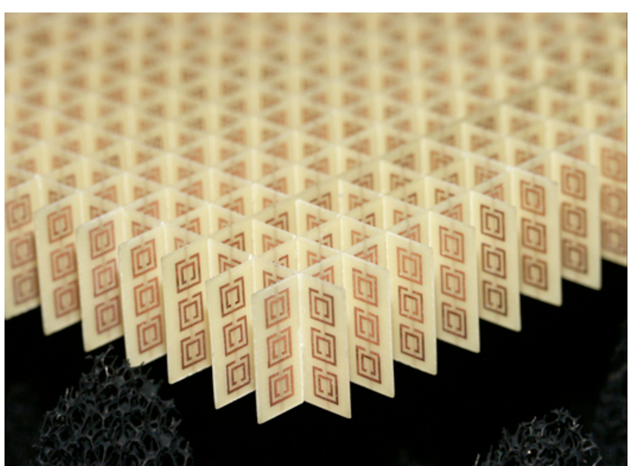
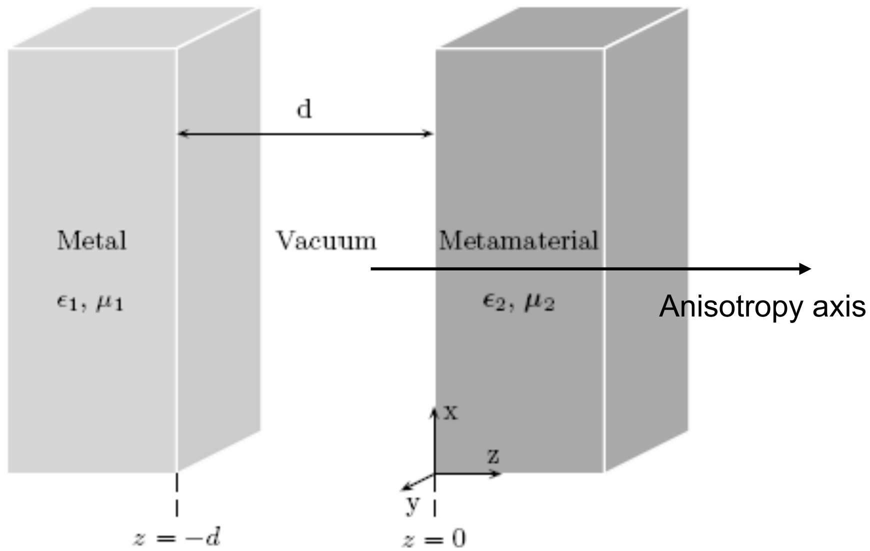


Examples of uniaxial anisotropy in stacked MMs

Anisotropy: Uniaxial MMs



$$\epsilon_{ij} = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \; ; \; \mu_{ij} = \begin{bmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{xx} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}$$

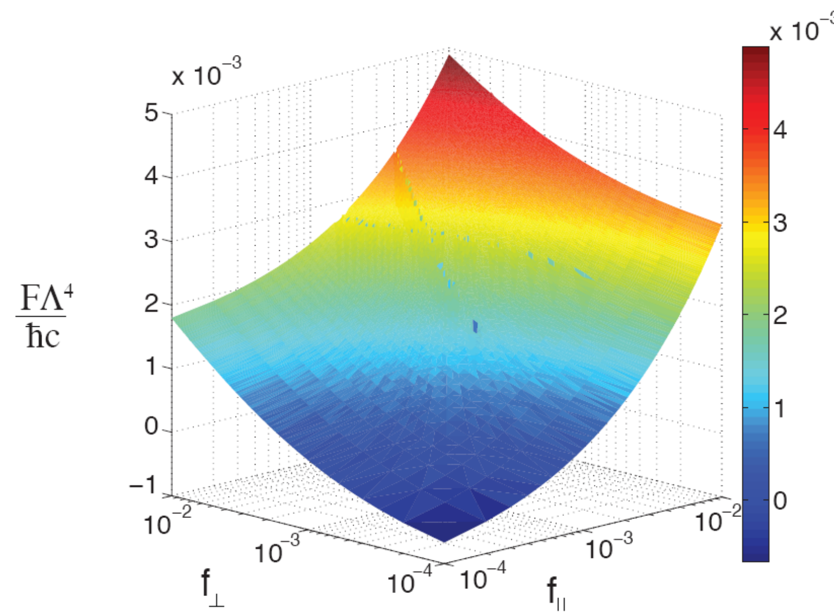
Anisotropy produces polarization mixing (non-diagonal reflection matrices)

$$\epsilon_{xx}(\omega) = \epsilon_{yy}(\omega) = 1 - (1 - f_x)\frac{\Omega_{e,x}^2}{\omega^2 - \omega_{e,x}^2 + i\gamma_{e,x}\omega} - f_x \frac{\Omega_{D,x}^2}{\omega^2 + i\gamma_{D,x}\omega},$$

$$\epsilon_{zz}(\omega) = 1 - (1 - f_z)\frac{\Omega_{e,z}^2}{\omega^2 - \omega_{e,z}^2 + i\gamma_{e,z}\omega} - f_z \frac{\Omega_{D,z}^2}{\omega^2 + i\gamma_{D,z}\omega},$$

$$\mu_{xx}(\omega) = \mu_{yy}(\omega) = 1 - \frac{\Omega_{m,x}^2}{\omega^2 - \omega_{m,x}^2 + i\gamma_{m,x}\omega},$$

$$\mu_{zz}(\omega) = 1 - \frac{\Omega_{m,z}^2}{\omega^2 - \omega_{m,z}^2 + i\gamma_{m,z}\omega}$$



Anisotropy does not help repulsion

(Rosa, DD, Milonni, PRA 2008)

EMA: correct model for μ

Drude-Lorentz for permeability is wrong. The correct expression that results in EMA from Maxwell's equations is

$$\boxed{\mu_{\text{eff}}(\omega) = 1 - f\frac{\omega^2}{\omega^2 - \omega_m^2 + 2i\gamma_m\omega}}$$

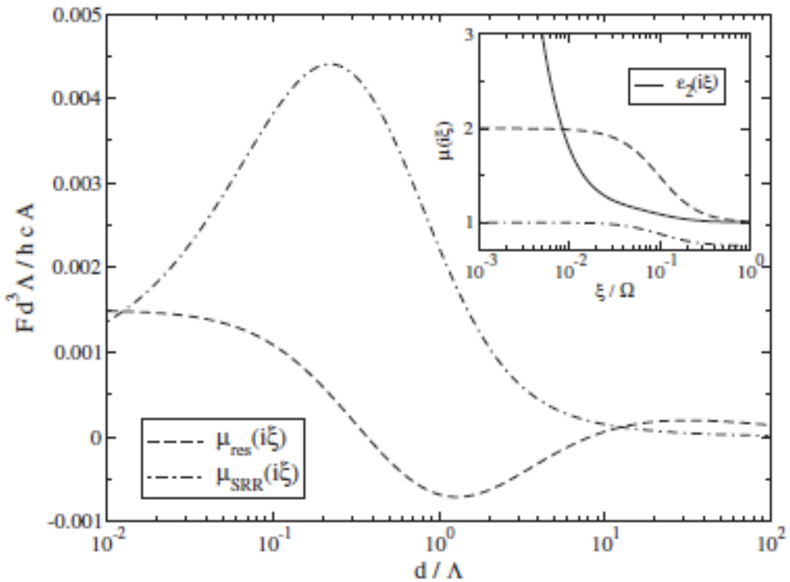
(Pendry 1999)

The appearance of the ω^2 factor in the numerator is very important:

Although close to the resonance this behaves in the same way as the Drude-Lorentz EMA permeability, it has a completely different low-frequency behavior

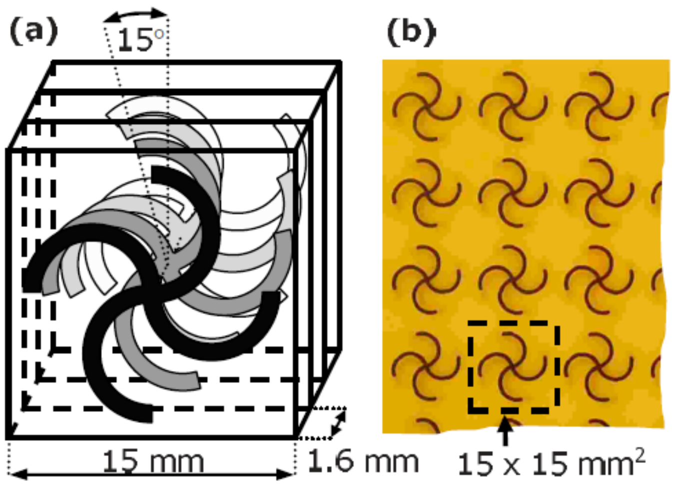
$$\mu_{\text{eff}}(i\xi) < 1 < \epsilon_{\text{eff}}(i\xi)$$

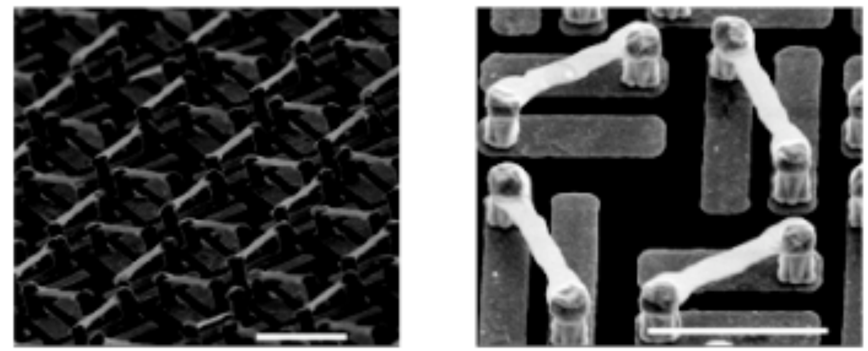
No Casimir repulsion!



Other Casimir MMs: chirality

The chirality of a MM is defined by the chirality of its unit cell





In a chiral medium, the constitutive relations mix electric and magnetic fields

$$\mathbf{D}(\mathbf{r},\omega) = \epsilon(\omega)\mathbf{E}(\mathbf{r},\omega) - i\kappa(\omega)\mathbf{H}(\mathbf{r},\omega)$$
$$\mathbf{B}(\mathbf{r},\omega) = i\kappa(\omega)\mathbf{E}(\mathbf{r},\omega) + \mu(\omega)\mathbf{H}(\mathbf{r},\omega)$$

dispersive chirality: $\kappa(\omega) = \dfrac{\omega_k \omega}{\omega^2 - \omega_{\kappa R}^2 + i\gamma_k \omega}$

Repulsion and chiral MMs

In chiral MMs the reflection matrix is non-diagonal (mixing of E and H fields).

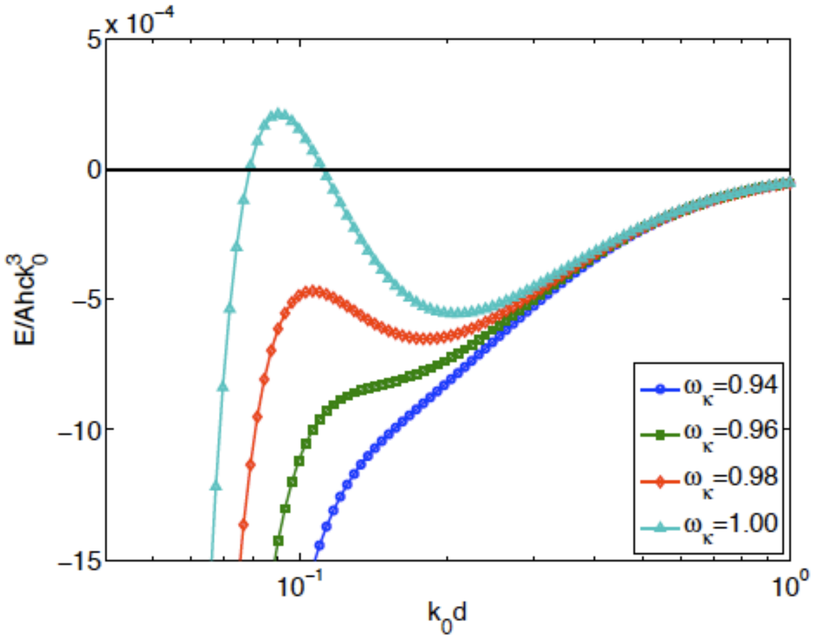
The integrand of the Casimir-Lifshitz force between two identical chiral MMs has the form:

$$F = \frac{(r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} - 2(r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}{1 - (r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} + (r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}$$

One might achieve repulsive Casimir forces with strong chirality (i.e., large values of r_{sp})

Same-chirality (SC) materials: repulsion

Opposite-chirality (OC) materials: repulsion



Soukoulis et al, PRL 2009

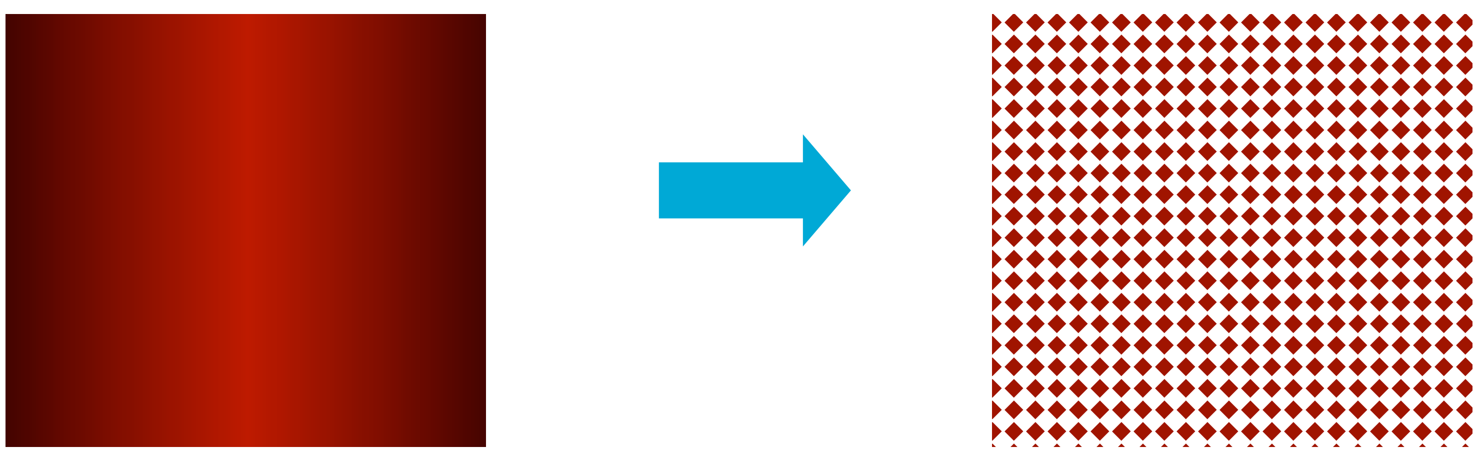
Beyond EMA

Everything discussed so far is based on the assumption that the effective medium approximation (EMA) holds. We recall that this amounts to treating the MM in the "long-wavelength approximation", i.e., field wavelengths much larger than the typical size of the unit cell of the MM.

How to calculate Casimir forces when EMA does not hold?
Can one trust predictions of Casimir repulsion with MMs based on EMA?

Homogeneous medium

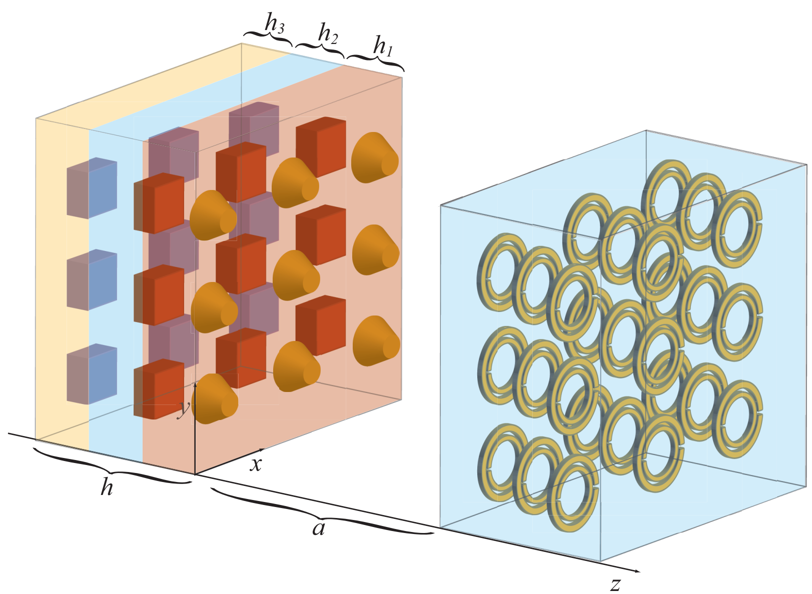
Non-homogeneous medium



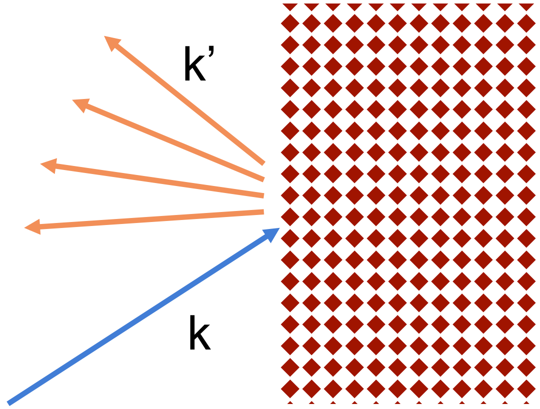
EMA

beyond EMA

Exact method: Scattering theory



The Casimir force still may be described in terms of reflections (scattering theory)



Symbolically, we may write the Casimir energy as

$$\boxed{\frac{E(d)}{A} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \log\det\left[1 - \mathcal{R}_1 e^{-\mathcal{K}d} \mathcal{R}_2 e^{-\mathcal{K}d}\right]}$$

where $\mathcal{R}_i \equiv \mathcal{R}_i(\mathbf{k}_{||}, \mathbf{k}'_{||}, p, p', i\xi)$

Solving for the reflection matrix

The reflection matrix can be obtained with standard methods of numerical electromagnetism. One way is to solve Maxwell equations for the transverse fields

$$-ik\frac{\partial \mathbf{E}_t}{\partial z} = \nabla_t\left[\chi\hat{e}_3\cdot\nabla\times\mathbf{H}_t\right] - k^2\mu\hat{e}_3\times\mathbf{H}_t$$
$$-ik\frac{\partial \mathbf{H}_t}{\partial z} = -\nabla_t\left[\zeta\hat{e}_3\cdot\nabla\times\mathbf{E}_t\right] + k^2\epsilon\hat{e}_3\times\mathbf{E}_t$$

Assuming a two-dimensional periodic structure, we have

$$\mathbf{E}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}}\sum_{m,n}\mathcal{E}_{m,n}\ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$
$$\mathbf{H}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}}\sum_{m,n}\mathcal{H}_{m,n}\ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

where

$$\epsilon(x,y) = \sum_{m,n}\epsilon_{m,n}\ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$
$$\mu(x,y) = \sum_{m,n}\mu_{m,n}\ \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

Exact reflection matrix

One can then write the equations for the transverse fields as

$$\boxed{-ik\frac{\partial \Psi_{m'n'}}{\partial z} = \sum_{mn} H_{m'n',mn}\Psi_{mn}}$$

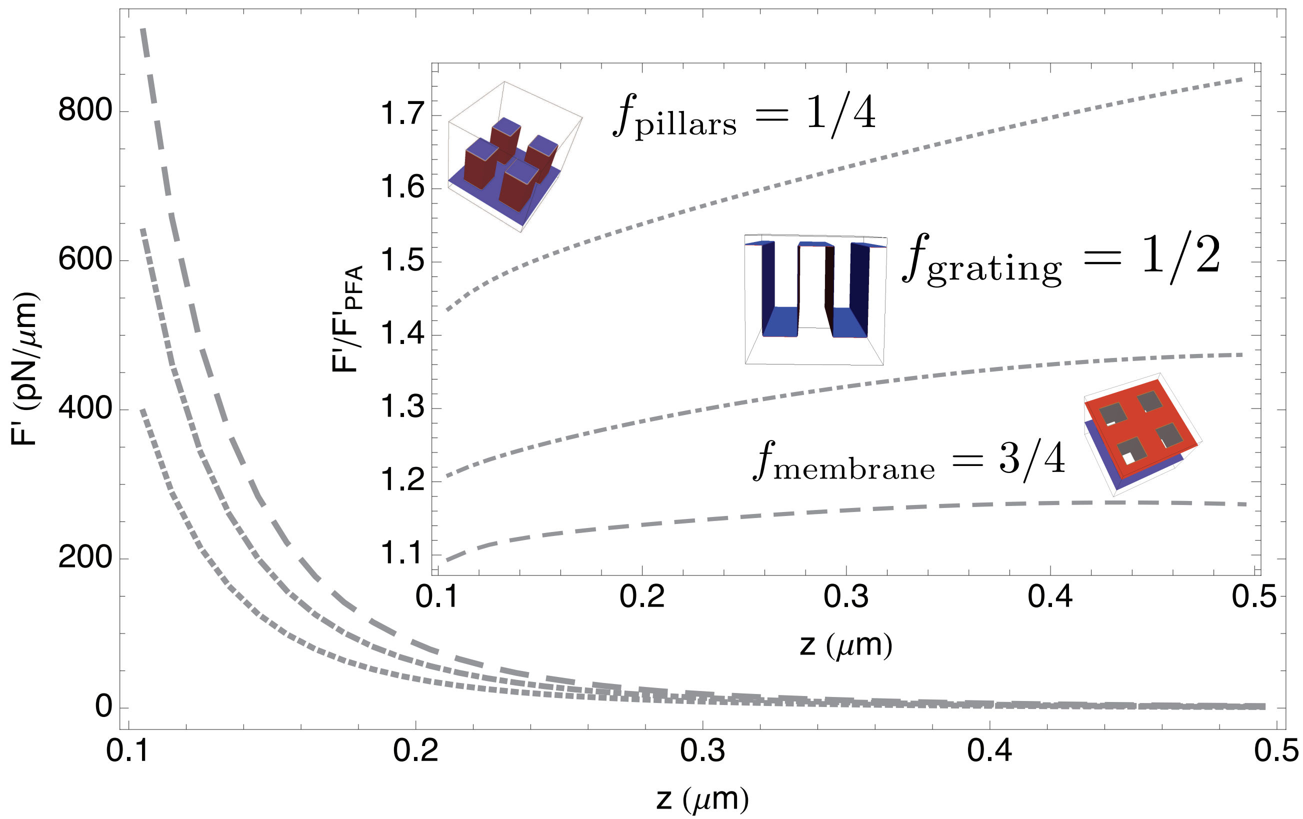
$$\Psi_{mn} = \begin{bmatrix} \mathcal{E}^x_{mn} \\ \mathcal{E}^y_{mn} \\ \mathcal{H}^x_{mn} \\ \mathcal{E}^y_{mn} \end{bmatrix} = \begin{bmatrix} \Psi^1_{mn} \\ \Psi^2_{mn} \\ \Psi^3_{mn} \\ \Psi^4_{mn} \end{bmatrix}$$

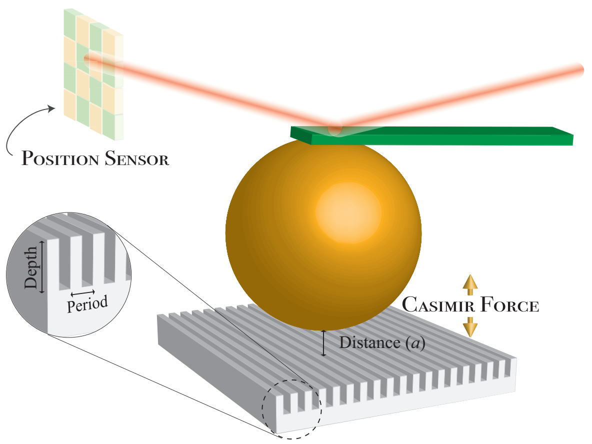
Here H is a complicated matrix, that encapsulated the coupling of modes in the periodic structure.

By numerically solving this equation and imposing the proper boundary conditions of the field on the vacuum-metamaterial interphase (RCWA or S-matrix techniques), one can find the reflection matrix of the MM.

2D periodic structures - finite T

Casimir force between a Au sphere and Si pillars/grating/membrane @ T=300 K





$$R = 50\mu m$$

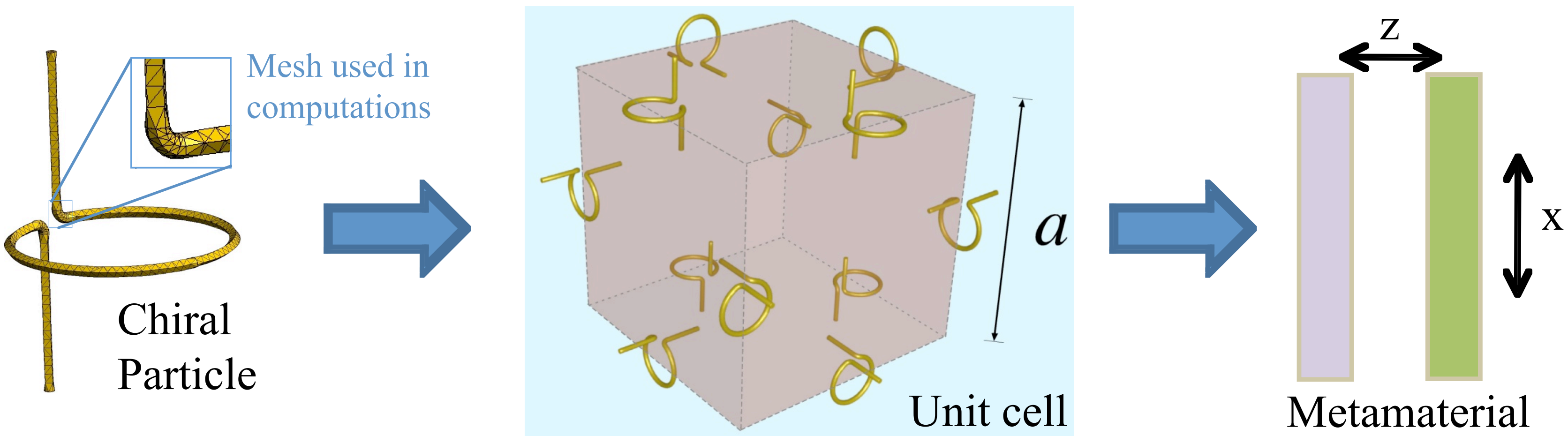
$$\text{period} = 400 \text{ nm}$$

$$\text{depth} = 1070 \text{ nm}$$

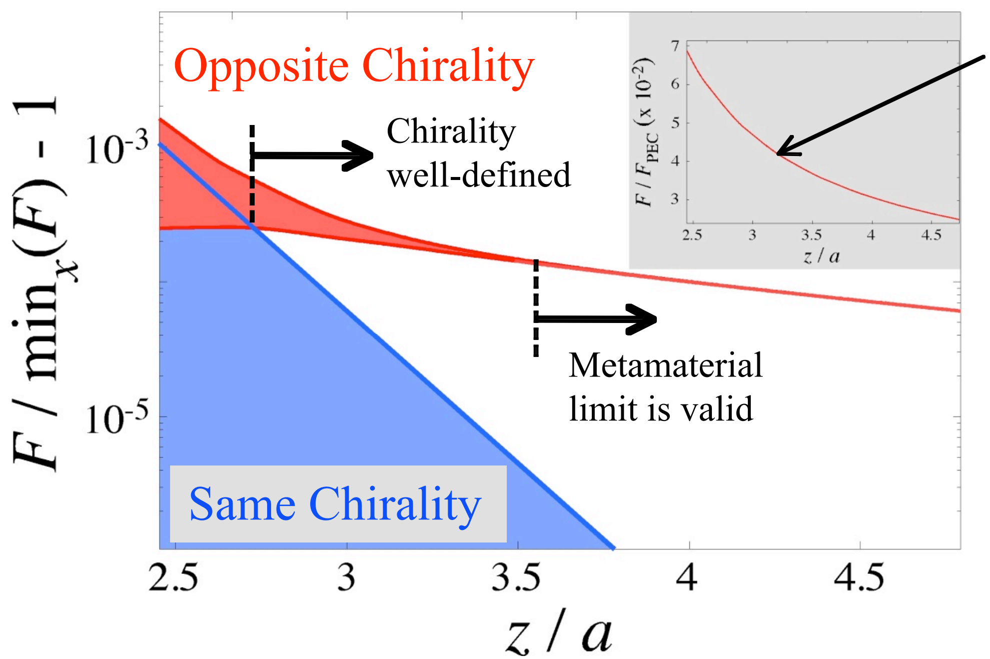
Davids, Intravaia, Rosa, DD, arXiv:1008.3580

Related techniques in 1D gratings at T=0K developed by Guérout, Lambrecht, Chan, et al. arXiv:1009.3487

Chiral metamaterials - T=0



Effect of inhomogeneity across displacements x



Total force relative to parallel metal plates

"repulsive" effect (force reduction) of chirality is one ten-thousandth of this!

Conclusion

In the regime where the chiral metamaterial limit is valid, the effect is **too small** to be observable.

McCauley, Johnson, DD, et al, PRB 2010

Remarks: MMs and Casimir

- Metamaterials offer an interesting possibility for Casimir force manipulation: engineered optical response, (maybe) broadband, dynamic control.

- Several proposals for MM-based Casimir force use effective medium approximation. Their predictions have to be carefully checked since EMA breaks down for electromagnetic fluctuations with wavelengths comparable to metamaterial feature sizes.

- Casimir repulsion in vacuum-separated metallic/dielectric metamaterial structures seems hard to achieve. It is certainly impossible in geometries that are effectively one-dimensional (Casimir stability considerations).

References

- Casimir-Lifshitz theory and magneto-dielectric metamaterials

Henkel and Joulain, EPL **72**, 929 (2005)

Leonhardt and Philbin, NJP **9**, 254 (2007)

da Rosa, DD, Milonni, PRL **100**, 183602 (2008)

da Rosa, DD, Milonni, PRA **78**, 032117 (2008)

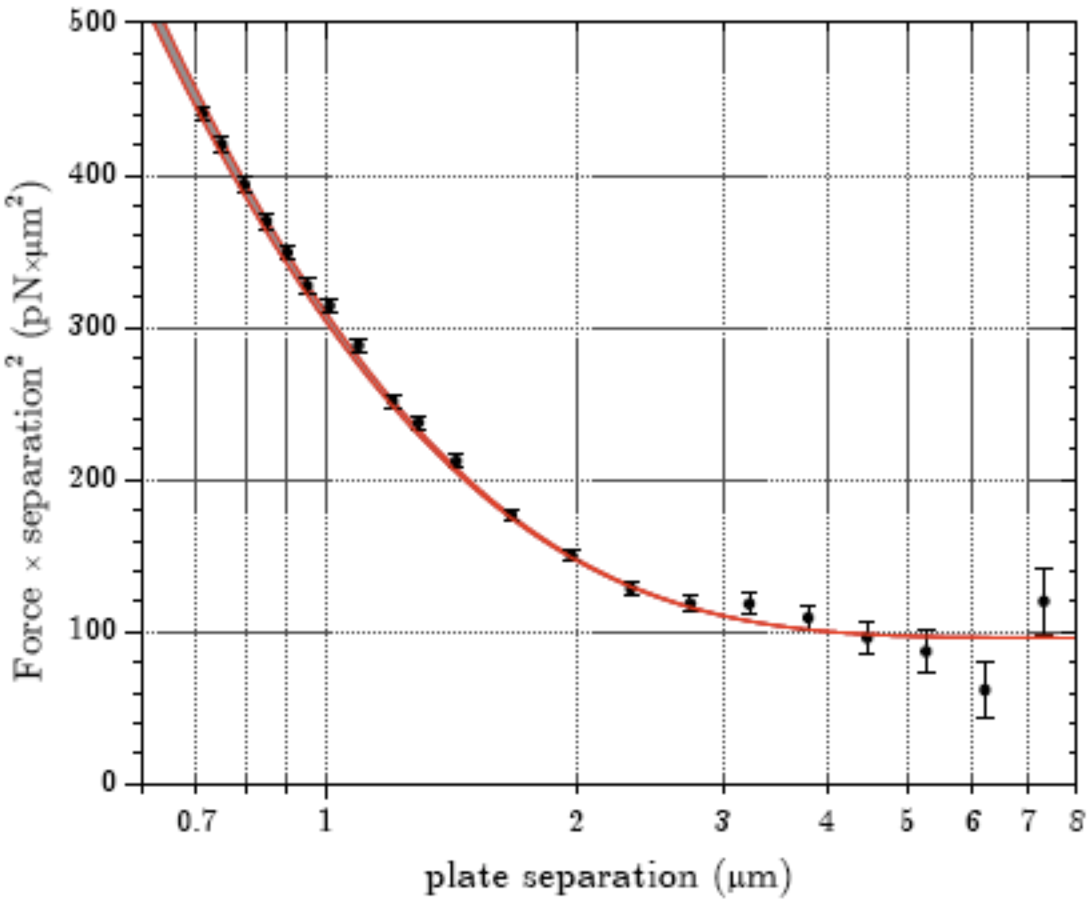
- Numerical techniques for Casimir interactions with metamaterials

McCauley et al, PRB **82**, 165108 (2010)

Davids, Intravaia, Rosa, DD, arXiv:1008.3580

New results (under review)

First observation of the thermal Casimir force

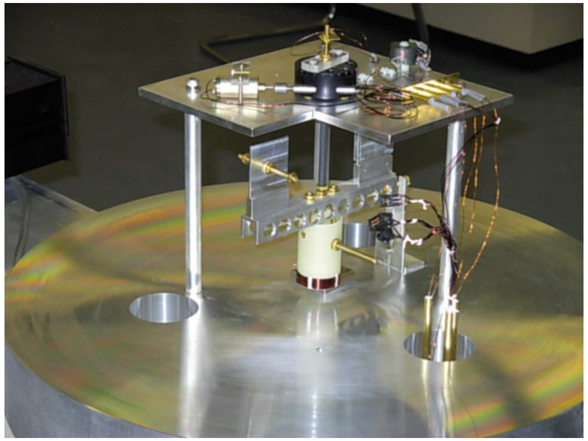


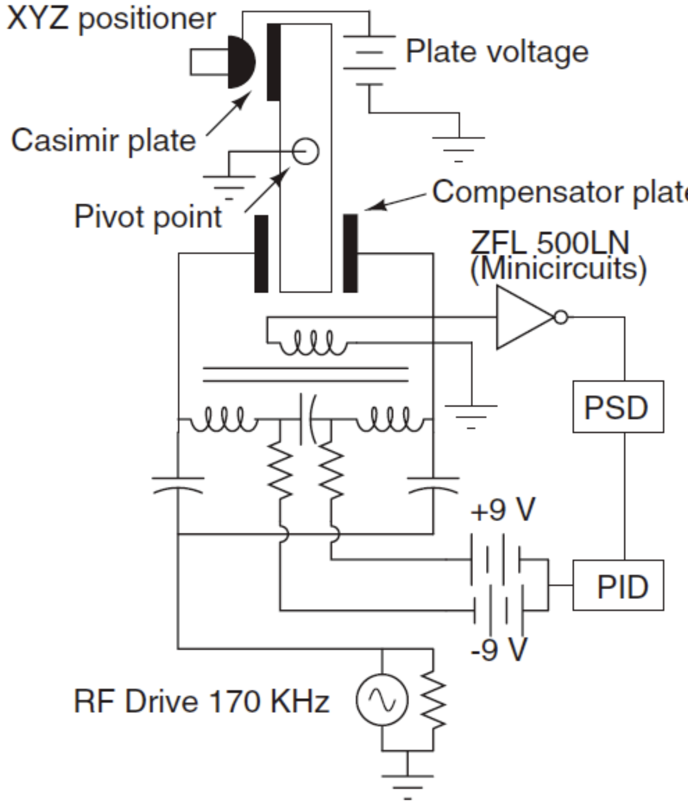
Sushkov (Yale), Kim (Seattle), DD (LANL), and Lamoreaux (Yale)

Torsional Pendulum Set-up

Los Alamos NATIONAL LABORATORY

* Upgrade of Lamoreaux's 1997 experiment





An imbalance in capacitance is amplified and sent to a phase sensitive detector (PSD), which generates error signals.

A proportional integro-differential (PID) controller provides a feedback correction voltage $S_{\rm PID}(d, V_a)$ to the compensator plates, restoring equilibrium.

$$F \propto (S_{\rm PID} + 9V)^2 \approx (9V)^2 + 2S_{\rm PID} \times 9V$$

The correction voltage is the physical observable, and it is proportional to the force between the Casimir plates

Typical Casimir Measurement

Los Alamos NATIONAL LABORATORY

$$S_{\mathrm{PID}}(d, V_a) = S_{\mathrm{dc}}(d \to \infty) + S_a(d, V_a) + S_r(d)$$

force-free component of signal at large separations

electrostatic signal in response to an applied external voltage

residual signal due to distance-dependent forces, e.g. Casimir

The electrostatic signal between the spherical lens and the plate, in PFA ($d \ll R$), is

$$S_a(d, V_a) = \pi\epsilon_0 R(V_a - V_m)^2/\beta d$$

β force-voltage conversion factor

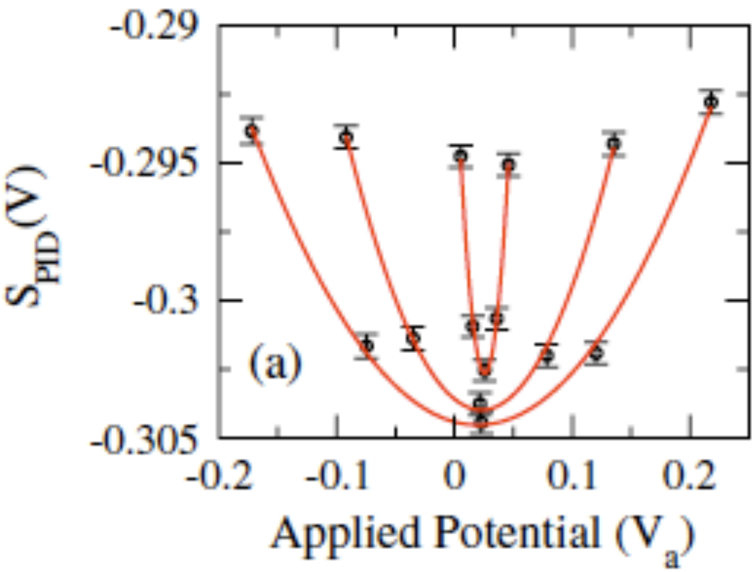
This signal is minimized ($S_a = 0$) when $V_a = V_m$, and the electrostatic minimizing potential V_m is then defined to be the contact potential between the plates.

"Parabola" measurements

Calibration routine (Iannuzzi et al, PNAS 04)

A range of plate voltages V_a is applied, and at a given nominal absolute distance the response is fitted to a parabola

$$S_{\mathrm{PID}}(d, V_a) = S_0 + k(V_a - V_m)^2$$



Fitting parameters

$k = k(d)$ ⟶ voltage-force calibration factor + absolute distance

$V_m = V_m(d)$ ⟶ distance-dependent minimizing potential

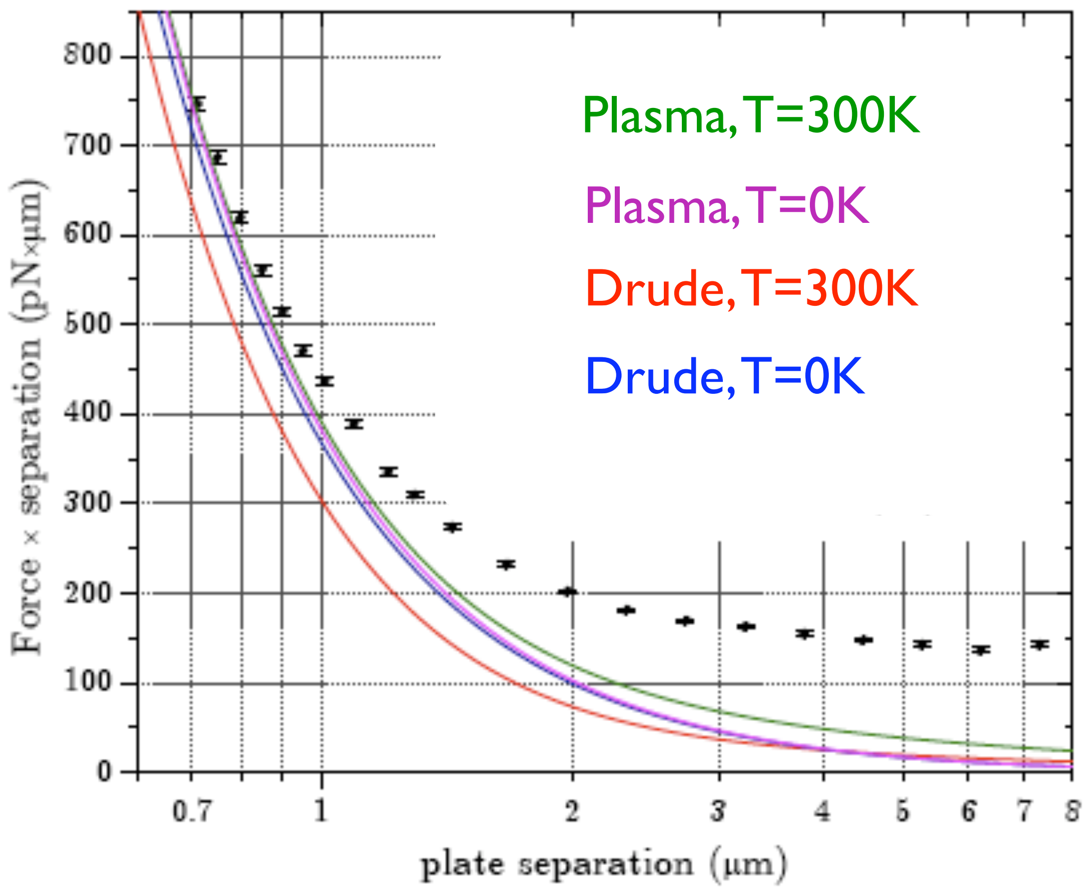
$S_0 = S_0(d)$ ⟶ force residuals: electrostatic + Casimir + non-Newtonian gravity + ...

This procedure is repeated at decremental distances, from 7 um down to 0.7 um, completing a single experimental run.

In the experiment $V_m = V_m(d)$ is almost constant (0.2 mV variation in the whole range)

Force Residuals

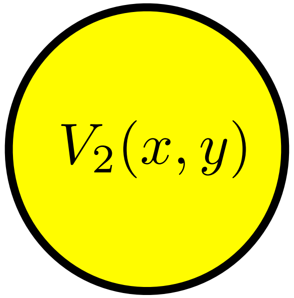
Residuals from Coulomb force obtained from the value of the PID signal at the minima of each parabola,



In the experiment, these force residuals are *too large* to be explained just by the Casimir-Lifshitz force between the Au plates.

Electrostatic Patch Effects

Sphere-plane geometry:



To compute the patch effect in the sphere-plane configuration we use PFA for the curvature effect $(d \ll R)$ but leave kd arbitrary

$$\nabla^2 V(x,y,z) = 0$$

$$F_{sp}(d) = 2\pi R \langle U_{pp}(d) \rangle = \frac{\epsilon_0 R}{16} \int_0^\infty dk \, \frac{k^2 e^{-kd}}{\sinh(kd)} \left[C_{1,k} + C_{2,k}\right]$$

$$\boxed{V(z=0) = V_1(x,y)}$$

Different models to describe surface potential fluctuations:

- $C_{1,k} = C_{2,k} = V_0^2$ for $k_{\min} < k < k_{\max}$

$$F_{sp} = \frac{4\pi\epsilon_0 V_{\rm rms}^2 R}{k_{\max}^2 - k_{\min}^2} \int_{k_{\min}}^{k_{\max}} dk \frac{k^2 e^{-kd}}{\sinh(kd)}$$

- $$\mathcal{R}(r) = \begin{cases} V_0^2 & \text{for } r \leq \lambda, \\ 0 & \text{for } r > \lambda. \end{cases}$$

$$F_{sp} = 2\pi\epsilon_0 R \int_0^\infty du \; u \frac{J_1(u)}{e^{2ud/\lambda} - 1}$$

In the limit of large patches $(kd \ll 1)$:

$$\boxed{F_{sp}(d) = \pi\epsilon_0 R \, \frac{V_{\rm rms}^2}{d}}$$

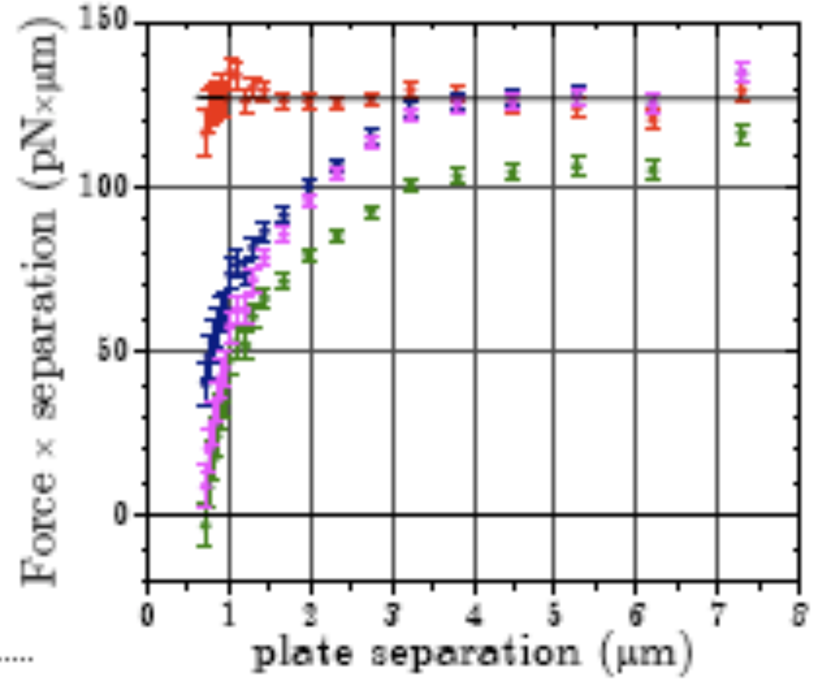
Understanding elec. residuals

We fit the data for the *residual force at the minimizing potential* with a force of equal to Casimir + patch effect

$$F_r(d) = F_C(d) + \pi\epsilon_0 R \frac{V_{\rm rms}^2}{d}$$

Drude, T=300K

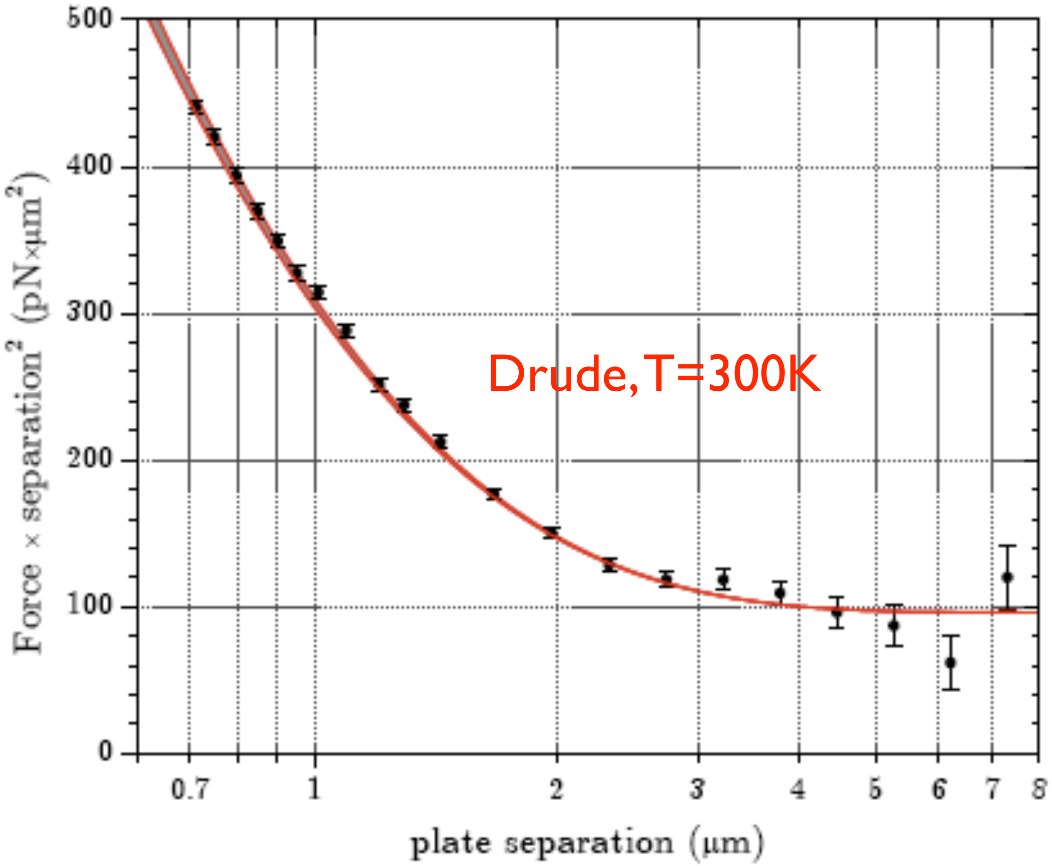
Drude, T=0K



Plasma, T=300K

Plasma, T=0K

Thermal Casimir force



$$F_C^{(T)}(\text{Drude}) = \frac{\xi(3)}{8}\,\frac{R k_B T}{d^2}$$

Quality of fits:

	χ^2_{red}	$V_{\text{rms}}(\text{m}V)$
Drude, T=300K	1.04	5.4
Plasma, T=300K	32	3.0
Drude, T=0K	23	4.0
Plasma, T=0K	43	3.6

Remarks:

- Experiment rules out the plasma model in the separation range 0.7um to 7um, and confirms the Drude model

- Thermal correction to the Casimir force demonstrated.

- Electrostatic residuals modeled as due to large electrostatic patches

$$F_r^{\rm patches} \propto R \frac{V_{\rm rms}^2}{d} \qquad (\lambda_{\rm P} \gg d)$$

- Would be interesting to do Kelvin probe microscopy of the used samples. Must detect potential variations on the 0.1 mV range.